Simple methods for calculating bend geometry in a tapered tube

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Introduction

The transmission line (TL) is a type of speaker enclosure that has received relatively little attention by audio designers and even less from the audio consumer. This is not for a lack of advantages as it has been shown that they can provide very high quality sound reproduction, especially when reproducing frequencies below 120Hz. Even the popular rise of the home theatre has not trickled down to a corresponding increase in the use of the TL.

One reason for this lack of popularity has been the physical size of the typical TL enclosure. Many TL's can approach a length of 8 feet and when constructed with a 12" diameter driver, the entire speaker can easily take up the same cubic space as a normal 3-cushion sofa. While this is not usually a disadvantage to some of the more enthusiastic audio consumers, there are many living spaces that cannot actually contain such a large object. Even in venues that can hold a TL, placement of the unit for optimum acoustic effect can be next to impossible for those spaces that are only just large enough.

The second reason is aesthetic. Aside from the overly enthusiastic, there are very few audio consumers who will tolerate the existence of a large pipe in their living quarters. In the typical home installation, sonic compromises are often preferable to visual obstructions.

Although it is not usually possible to change the length of any given TL, it is possible to change the configuration it takes within any given 3D space. One generally accepted method for re-configuring the TL involves "folding" it so that the dimensions of the enclosure are closer to that of a cube than a cylinder. Instead of extending in a straight line, the enclosure topology is very much like a section of a maze.

Rather than discussing various fold configurations, this paper will explain simplified methods for designing the necessary bends.

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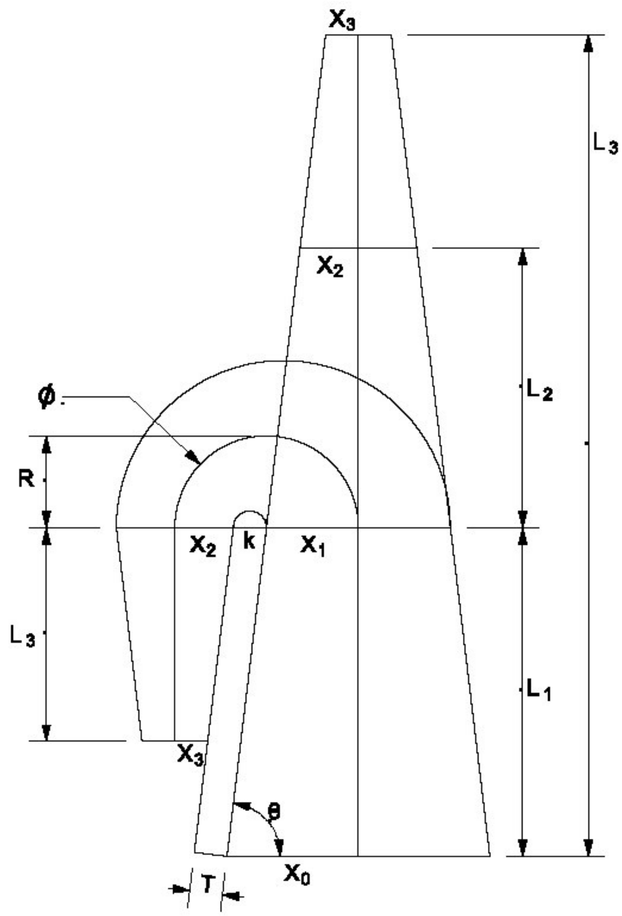
Analysis

The diagram shows the problem normally encountered when determining the bend geometry in a tapered transmission line.

For any given bend, the effective length of the bend determines the width of the line at the end of the bend. However, the width of the line at that point also determines the radius of the bend which itself is proportional to the length of the bend. The solution lies in making the width of the line at the end of the bend dependent on known factors. In our simplified case, these factors are:

1) Total line length
2) Starting line width
3) End width of line
4) Material thickness
5) Line length before the bend
6) Line width at bend start

Once the width at the end of the bend is known, calculating the radius of the bend is easy and fabrication can follow.



Assumptions

Development of our method would not be possible without narrowing down the scope in which we can apply our solution. While it is possible to derive methods for solving more topologies, we do not discuss that here. The required constraints are as follows:

1) The cross-sectional profile of the TL is a shape that has at least two axes of symmetry (rectangle, oval)
2) The diagram relates to dimensions along one of the axes of symmetry.
3) The taper (rate of change of cross-section) is constant
4) Bends do not meet straight sections of the line at a tangent.
5) The bend is only an approximation of a spiral

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Calculations

The constant slope of the taper is required to derive section widths are various point in the TL.

$$\Delta = \tan \Theta = \frac{L_3}{(X_0 - X_3)}$$

Wall thickness varies with the taper and is a constant > 0 for real world materials.

$$k = \frac{T}{\sin \Theta}$$

The width at the end of the first section (and the start of the bend) depends on the section length and initial line width

$$\Delta = \frac{L_1}{(X_0 - X_1)}$$

$$X_1 = X_0 - \frac{L_1}{\Delta}$$

The second section length, the second section end width and the bend radius are inter-related and can be derived from the swept angle of the bend and the slope of the taper.

$$\Delta = \frac{L_2}{(X_1 - X_2)}$$

$$\frac{L_2}{(X_1 - X_2)} = \frac{L_1}{(X_0 - X_1)}$$

$$L_2 = \frac{L_1\ (X_1 - X_2)}{(X_0 - X_1)}$$

L_2 is also the same as the average length of the bend

$$L_2 = \Phi\ R$$

where R is the averaged radius at the bend

$$R = \frac{(X_1 + X_2 + k)}{2}$$

substituting for R gives

$$L_2 = \frac{\Phi\ (X_1 + X_2 + k)}{2}$$

so we get the equivalent expression

$$\frac{L_1\ (X_1 - X_2)}{(X_0 - X_1)} = \frac{\Phi\ (X_1 + X_2 + k)}{2}$$

isolating X_2 for easy solution we get the following sequence

$$L_1\ X_1 - L_1\ X_2 = \frac{\Phi\ (X_1 + k)\ (X_0 - X_1) + \Phi\ X_2\ (X_0 - X_1)}{2}$$

$$L_1\ X_1 - \frac{\Phi\ (X_1 + k)\ (X_0 - X_1)}{2} = L_1\ X_2 + \frac{\Phi\ X_2\ (X_0 - X_1)}{2}$$

swap sides and rearrange for a solution to X_2

$$X_2 = \frac{2\ L_1\ X_1 - \Phi\ (X_1 + k)\ (X_0 - X_1)}{2\ L_1 + \Phi\ (X_0 - X_1)}$$